

Homework 2

1. Using either the properties of limits and/or the squeezing principle, find the following limits:

$$(a) \quad \lim_{n \rightarrow \infty} \frac{3n^4 + (-1)^n n^2 + 1}{4n^4 + 3n^3 + 2n}$$

$$(b) \quad \lim_{n \rightarrow \infty} \frac{n^2 + 2}{n^2 \sqrt{n} + 1}$$

$$(c) \quad \lim_{n \rightarrow \infty} \frac{2^n + 3}{3^n + 2}$$

$$(d) \quad \lim_{n \rightarrow \infty} \sqrt[n]{3}$$

$$(e) \quad \lim_{n \rightarrow \infty} (2^n + 3^n)^{1/n}$$

2. Let $x_1 = 1$, and $x_{n+1} = \sqrt{2 + x_n}$, $n \geq 1$. Show that (x_n) converges and find its limit.

3. Let $x_1 = 1, x_2 = 2, x_{n+1} = \frac{1}{4}(x_n + x_{n-1})$, $n \geq 2$. Prove that x_n is convergent.

4. Prove that

$$x_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$$

converges.