

Review 1

1. Evaluate the following limits

$$\lim_{x \rightarrow 1^+} \frac{x^2 + 3x - 10}{x - 2}$$

$$\lim_{x \rightarrow 1^+} \frac{x^2 + 3x - 10}{x - 1}$$

$$\lim_{x \rightarrow 2^+} \frac{x^2 - 4x + 4}{x - 2}$$

$$\lim_{x \rightarrow 3^-} \frac{x^2 - 4x + 3}{x - 3}$$

$$\lim_{x \rightarrow \infty} \frac{x^3 + 3x^2 + 2}{x^2 + 4x - 1}$$

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 3x^2 + 2}{x^2 + 4x - 1}.$$

2. (a) Find the numbers a and b such that the function

$$f(x) = \begin{cases} 3x^2 + ax + 4 & \text{if } x < 2, \\ 5 & \text{if } x = 2, \\ \frac{x+a}{x-1}, & \text{if } x > 2 \end{cases}$$

is continuous everywhere.

(a) Find the numbers a and b such that the function

$$f(x) = \begin{cases} ax^3 + bx + 4 & \text{if } x < -1, \\ 2 & \text{if } x = -1, \\ \frac{ax+b}{x+2}, & \text{if } x > -1 \end{cases}$$

is continuous everywhere.

3. Using the definition only, find the derivatives of the following functions at the specified points

$$f(x) = x^3 \text{ at } x = 2$$

$$f(x) = x^2 \text{ at } x = -2$$

$$f(x) = \frac{1}{x} \text{ at } x = 1$$

$$f(x) = \frac{1}{x+1} \text{ at } x = 1$$

$$f(x) = \sqrt{x} \text{ at } x = 4$$

$$f(x) = \sqrt{x+3} \text{ at } x = 6.$$

4. Using the definition only, find the derivatives of the following functions

$$f(x) = 2x^2 + 3x + 4$$

$$f(x) = x^3 + x$$

$$f(x) = \sqrt[3]{x}$$

$$f(x) = \sqrt[3]{x^2 + 2x}.$$

5. Using properties of derivatives find the derivatives of the following functions

$$f(x) = e^{-x^2+3x}$$

$$f(x) = e^{\sec x}$$

$$f(x) = x \sin 5x$$

$$f(x) = (x + 1)^3 \tan 3x$$

$$f(x) = \frac{\sqrt{1-3x}}{x+2}$$

$$f(x) = \frac{1 + e^{2x}}{1 - e^{3x}}.$$

6. Using properties of derivatives find the derivatives of the following functions

$$f(x) = \sqrt{\frac{x^2 + 3}{x^2 - 5}}$$

$$f(x) = \sqrt[3]{\frac{x}{2x - 3}}$$

$$f(x) = xe^x \sin x$$

$$f(x) = \frac{x \cos x}{3x + 1}$$

$$f(x) = \sqrt{x} \sin^{-1} x$$

$$f(x) = \tan^{-1}(3x + 2).$$

7. Find dy/dx if

$$x^2 + y^3 = 20$$

$$xy - x = y + 2$$

$$\cos xy = 1 - x^2$$

$$e^{xy} + y^2 = 25.$$