

Review 2

1. Find the equation of the tangent line to the graph of the function at the specified point

$$f(x) = \frac{x+2}{x-3} \text{ at } x = 5,$$
$$g(x) = \frac{3x+4}{4x-1} \text{ at } x = 2.$$

2. Find the equation of the line tangent to the curve at the specified point

$$x^2 + y^2 = 4x + 5y \text{ at } (4, 5),$$
$$x \tan^{-1} y = x^2 + y \text{ at } (0, 0).$$

3. One end of a rope is fastened to a boat and the other end is wound around a windlass located on a dock at a point 4m above the level of the boat. If the boat is drifting away from the dock at a rate of 2m/min, how fast is the rope unwinding at the instant when the length of the rope is 5m?

4. A rock is dropped into a lake and an expanding circular ripple results. When the radius of the ripple is 8 in, the radius is increasing at a rate of 3 in/s. At what rate is the area enclosed by the ripple changing at this time?

5. Find the absolute extrema of the given functions on the specified intervals

$$f(x) = \frac{\ln \sqrt{x}}{x}, \text{ on } [1, 3],$$
$$g(x) = 3t^5 - 20t^3, \text{ on } [-1, 2],$$
$$h(x) = e^{-x} \sin x, \text{ on } [0, 2\pi],$$
$$k(x) = x^3 - 6x^2 + 9x + 1, \text{ on } [0, 2].$$

6. Sketch the graphs of the following functions

$$f(x) = x^4 - 4x^3 + 3,$$
$$g(x) = x^3 - 3x^2 + 2,$$
$$h(x) = \frac{x}{x^2 + 1},$$
$$k(x) = e^{-x^2}.$$

7. Use l'Hospital's Theorem to compute the following limits

$$\lim_{x \rightarrow \infty} \frac{3x^3 + 4x^2 + 5x}{4x^3 + 3x + 1},$$

$$\lim_{x \rightarrow 1} \frac{x^8 - 1}{x^6 - 1},$$

$$\lim_{x \rightarrow 0} \frac{3^x - 2^x}{x^2 - x},$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2},$$

$$\lim_{x \rightarrow 0^+} x(\ln x)^2,$$

$$\lim_{x \rightarrow 0^+} \ln x \tan x,$$

$$\lim_{x \rightarrow 0^+} x^{\sin x},$$

$$\lim_{x \rightarrow \infty} x^{1/x}.$$