

Computing the surface integral is time consuming. We use instead the Divergence Theorem:

$$\iint_S \mathbf{F} \cdot \mathbf{N} dS = \iiint_R \operatorname{div} \mathbf{F} dV,$$

where in our situation S is the surface of the cylinder and R is the interior of the cylinder. We compute

$$\operatorname{div} \mathbf{F} = \frac{\partial}{\partial x} 0 + \frac{\partial}{\partial y} (xyz) + \frac{\partial}{\partial z} 0 = xz.$$

Switching to cylindrical coordinates we have

$$\begin{aligned} \iiint_R xz dx dy dz &= \int_0^{2\pi} \int_0^3 \int_0^5 (r \cos \theta) z r dz dr d\theta \\ &= \left(\int_0^{2\pi} \cos \theta d\theta \right) \left(\int_0^3 r^2 dr \right) \left(\int_0^5 z dz \right) \\ &= 0 \cdot \frac{27}{3} \cdot \frac{5^2}{2} = 0. \end{aligned}$$