This problem is very similar to Example 5 from 11.7 in the book, it is slightly easier than that example.

The domain of the function is the closed disk of radius 4 centered at the origin. We first find the critical points in the interior of the domain. For that we compute the partial derivatives of f and we set them equal to zero.

$$\frac{\partial f}{\partial x} = 3x = 0$$
$$\frac{\partial f}{\partial y} = 2y = 0.$$

We obtain $x = y = 0$, and indeed, the point $(0, 0)$ lies in the domain, so it is a critical point of f.

There are two ways of studying the function on the boundary. The first, as shown in the book, is to notice that on the boundary $y^2 = 4 - x^2$, so the function is equal to $3x^2 + 2(4 - x^2) = x^2 + 8$. And $-2 \leq x \leq 2$. So we have to find the extrema of the function $g(x) = x^2 + 8$ on the interval $-2 \leq x \leq 2$. Inside the interval the critical point of g is obtained from $g'(x) = 2x = 0$, it is $x = 0$. When $x = 0$, $y = \pm 2$. So we have to take into account the points $(0, 2)$, $(0, -2)$ on the boundary. We also have to take into acount the endpoints of the interval, namely -2 and 2, which give $y = 0$, and thus the points $(2, 0)$ and $(-2, 0)$.

Now we plug everything in: $f(0, 0) = 0$, $f(0, 2) = f(0, -2) = 8$, $f(2, 0) = f(-2, 0) = 12$. So the minimum of f is zero, and is attained at the origin, and the maximum is 12 and is attained at $(2, 0)$ and $(-2, 0)$.

Or, we could have looked at the problem on the boundary as a problem of maximizing a function with constraints, and use Lagrange multipliers. We have $f(x, y) = 3x^2 + 2y^2$ and $g(x, y) = x^2 + y^2 - 4$ and we find the extrema of f subject to the condition $g = 0$. The system $\nabla f = \lambda \nabla g$ and $g = 0$ is

$$6x = 3\lambda x, \quad 4y = 2\lambda y, \quad x^2 + y^2 = 4.$$

We solve and get $x = \pm 2, y = 0$ and $x = 0, y = \pm 2$. Now we substitute the points $(0, 0)$, $(\pm 2, 0)$ and $(0, \pm 2)$ as above, and we are done.