

The curl of \mathbf{F} is

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz^2 & xz^2 & 2xyz \end{vmatrix} = (2xz - 2xz)\mathbf{i} - (2yz - 2yz)\mathbf{j} + (z^2 - z^2)\mathbf{k} = \mathbf{0},$$

So the vector field is conservative.

We want to find a potential f such that $df = \mathbf{F}$. Then

$$f(x, y, z) = \int yz^2 dx = xyz^2 + c(y, z).$$

Differentiating we obtain $xz^2 + \partial c/\partial y = xz^2$ and $2xyz + \partial c/\partial z = 2xyz$, and hence c is constant; we can choose $c = 0$. Then $f(x, y, z) = xyz^2$.

Using the Fundamental Theorem of Calculus, we obtain

$$\int_C \mathbf{F} \cdot d\mathbf{R} = f(2, 2, 3) - f(1, 0, 1) = 36 - 0 = 36.$$