

We use Stokes' Theorem

$$\int_C \mathbf{F} \cdot d\mathbf{R} = \iint_S \text{curl } \mathbf{F} \cdot \mathbf{N} dS.$$

The curl of  $\mathbf{F}$  is

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix} = -\mathbf{i} - \mathbf{j} - \mathbf{k}.$$

The normal vector is orthogonal to the plane through  $(3, 0, 0)$ ,  $(0, 0, 2)$ ,  $(0, 6, 0)$ . Using the equation of a plane give by intercepts, the equation is

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{2} = 1,$$

or, to avoid working with fractions

$$2x + y + 3z = 1.$$

You can find the normal vector as in Example 2 at page 727, it is a multiple of  $(2, 1, 3)$ . This multiple should have norm 1, and point in the direction specified by the right-hand rule when the path is traveled. There are two vectors  $(2/\sqrt{14}, 1/\sqrt{14}, 3/\sqrt{14})$  and  $(-2/\sqrt{14}, -1/\sqrt{14}, -3/\sqrt{14})$  and we choose the second.

Write the surface (the triangle) as

$$z(x, y) = 2 - \frac{2}{3}x - \frac{1}{3}y,$$

and then the area element is

$$dS = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1 + \frac{4}{9} + \frac{1}{9}} = \frac{\sqrt{14}}{3}.$$

And the surface (triangle) covers the triangle  $T$  with vertices  $(0, 0, 0)$ ,  $(3, 0, 0)$ , and  $(0, 6, 0)$  in the  $xy$  plane. We compute

$$\iint_S \text{curl } \mathbf{F} \cdot \mathbf{N} dS = \iint_T (-1, -1, -1) \cdot (-2/\sqrt{14}, -1/\sqrt{14}, -3/\sqrt{14}) \frac{\sqrt{14}}{3} dx dy = 2\text{Area}(T) = 18.$$