First Exam

TO RECEIVE FULL CREDIT, YOU MUST SHOW ALL YOUR WORK. EACH PROBLEM IS WORTH 20 POINTS. IF A PROBLEM HAS SEVERAL PARTS, THE POINTS ARE SPLIT EVENLY AMONG THOSE PARTS. WRITE THE SOLUTIONS TO THE EXAM PROBLEMS ON ONE OR MORE SHEETS OF PAPER, AS NECESSARY, AND THEN UPLOAD THEM TO THE LINK THAT WILL BE MAILED TO YOU. THE DEADLINE FOR THE EXAM IS OCTOBER 6TH. THIS DEADLINE IS STRICTLY ENFORCED.

1. Let $f(x, y, z) = (x + 2y)^2 + (y + 2z)^2 + (z + 2x)^2$ and consider the point P_0 of coordinates $(-2, 1, 2)$. Find the direction in which the function increases most rapidly (the answer should be a unit vector) and compute the magnitude of the greatest rate of increase.

2. Let $\mathbf{R}(t) = (t\cos t)\mathbf{i} + (t\sin t)\mathbf{j} + (1 - t^2)\mathbf{k}$.
   (a) Find the unit tangent vector (do not compute the principal unit normal vector).
   (b) Find the curvature of the curve at $t = 0$.

3. Find the critical points of $f(x, y) = 3xy^2 - 2x^2y + 36xy$ and classify them as a relative maximum, a relative minimum, or a saddle point.

4. Find f_{xy} if
$$f(x, y, z) = \frac{xy + yz}{xz}.$$

5. Maximize $f(x, y) = 16 - x^2 - y^2$ subject to the constraint $x + 2y = 6$.