Second Exam

TO RECEIVE FULL CREDIT, YOU MUST SHOW ALL YOUR WORK. EACH PROBLEM IS WORTH 20 POINTS. IF A PROBLEM HAS SEVERAL PARTS, THE POINTS ARE SPLIT EVENLY AMONG THOSE PARTS. WRITE THE SOLUTIONS TO THE EXAM PROBLEMS ON ONE OR MORE SHEETS OF PAPER, AS NECESSARY, AND THEN UPLOAD THEM TO THE LINK THAT WILL BE MAILED TO YOU. THE DEADLINE FOR THE EXAM IS NOVEMBER 24TH. THIS DEADLINE IS STRICTLY ENFORCED.

1. Compute the integral

$$\iint_D xy dA$$

where D is the region bounded by $y = \sqrt{x}$, $y = 6 - x$, $y = 1$.

2. Find the volume of the solid D bounded from above by the surface $z = 9 - 4x^2 - 4y^2$ and below by the plane $z = 0$.

3. Compute the line integral

$$\int_C [2x^3y^2dx + x^4ydy]$$

where C is the path that travels first from $(1, 0)$ to $(0, 1)$ along the part of the circle $x^2 + y^2 = 1$ that lies in the first quadrant and then from $(0, 1)$ to $(-1, 0)$ along the line segment that connects the two points.

4. Compute the integral

$$\oint_C [(\cos x - 3y)dx + (2x - \sin y)dy),$$

where C is the closed curve that travels on the line segments from $(0, 0)$ to $(4, 0)$, from $(4, 0)$ to $(2, 1)$, and from $(2, 1)$ to $(0, 0)$.

5. Let $\mathbf{F} = 2x^3\mathbf{i} + 2y^3\mathbf{j} + 2z^3\mathbf{k}$ and let S be the surface consisting of the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$ and the disk $x^2 + y^2 \leq 1$, $z = 0$ in the xy plane. Find $\int_S \mathbf{F} \cdot \mathbf{N} dS$.