First Exam

1. The line $y = 6 - x$ intersects the parabola $y = \sqrt{x}$ at $(4, 2)$. We compute the integral as

$$\int_1^4 \int_1^{\sqrt{x}} xydydx + \int_4^5 \int_1^{6-x} xydydx = \frac{77}{8}.$$

2. We can use a double integral in cylindrical coordinates

$$\int_0^{2\pi} \int_0^1 (9 - 4r^2) r dr d\theta = \frac{81\pi}{8}.$$

3. The vector field is conservative and it has the potential $f(x,y) = \frac{1}{2}x^4y^2$. Using the fundamental theorem of calculus we obtain that the integral is $f(-1,0) - f(1,0) = 0$. You cannot apply Green's formula because the contour is not closed.

4. You can apply Green's formula in this case, and the integral becomes

$$\int\int_D (2-(-3))dxdy = \int\int_D 5dxdy.$$

The region D is a triangle with height 1 and base 4, its area is 2. The integral is therefore equal to $5 \times 2 = 10$.

5. The divergence of the vector field is $6(x^2 + y^2 + z^2)$. We switch to spherical coordinates, and write the integral as

$$\int_0^{2\pi} \int_0^{\pi/2} \int_0^1 6\rho^2 \cdot \rho^2 \sin\phi d\rho d\phi d\theta = \frac{12\pi}{5}.$$