Final Exam

TO RECEIVE FULL CREDIT, YOU MUST SHOW ALL YOUR WORK. EACH PROBLEM IS WORTH 10 POINTS. WRITE THE SOLUTIONS TO THE EXAM PROBLEMS ON ONE OR MORE SHEETS OF PAPER, AS NECESSARY, AND THEN UPLOAD THEM TO THE LINK THAT WILL BE MAILED TO YOU. THE DEADLINE FOR THE EXAM IS DECEMBER 8 AT 1 PM. THIS DEADLINE IS STRICTLY ENFORCED.

1. Find the 2-dimensional position vector $\mathbf{R}(t)$ given the velocity $\mathbf{V}(t) = \cos t\mathbf{i} + t^3\mathbf{j}$ and the initial position $\mathbf{R}(0) = \mathbf{i} - 2\mathbf{j}$.

2. Find the curvature of the curve $y = x^3$ at the point $(x, y) = (2, 8)$.

3. What is the rate of change of the function $f(x, y) = ye^{2\cos x} + \tan(x + 3y)$ at $(\frac{3\pi}{2}, \frac{\pi}{2})$ in the direction of the vector $(3, 4)$?

4. Find the critical points of $f(x, y) = x^3 - y^3 - 3x^2 + 12y + 1$ and classify them as a relative maximum, a relative minimum, or a saddle point.

5. Find the equation of the plane tangent to the surface $z = 10 - x^2 - y^2$ at the point $P_0(2, 2, 2)$.

6. Evaluate the integral

$$\iint_D (2y - x)dA$$

   where D is the region bounded by $y = x^2$ and $y = 2x$.

7. Evaluate

$$\iint_R \sin(x^2 + y^2)dxdy$$

   where R is the region inside the circle $x^2 + y^2 = \pi$.

8. Evaluate the triple integral

$$\iiint_T x^2 dV$$

   where T is the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$.

9. Compute the integral

$$\oint_C [(x^5 + y^3)dx - (x^3 + y^5)dy]$$

where C is the unit circle $x^2 + y^2 = 1$ traversed counterclockwise.

10. Use the divergence theorem to calculate the flux of the vector field $\mathbf{F} = x^3\mathbf{i} - 3x^2y\mathbf{j} + 6z\mathbf{k}$ through the surface of the sphere of radius 5 centered at the origin.