

Homework 5

1. In each of the following two problems, assume that the eigenvalues are real. Determine the form of the eigenfunctions and the algebraic equation that determines the eigenvalues. Estimate the eigenvalues that have large absolute values.

$$\begin{aligned}y'' - \lambda y &= 0, & y(0) &= 0, & y(\pi) + y'(\pi) &= 0, \\y'' - \lambda y &= 0, & y(0) &= y'(0) = 0, & y(1) &= 0.\end{aligned}$$

2. In each problem, transform the given equation into the form $(p(x)y')' + q(x)y = 0$:

$$\begin{aligned}x^2 y'' + x y' + (x^2 - \nu^2) y &= 0, & \nu & \text{ a real number,} \\(1 - x^2) y'' - x y' + \alpha^2 y &= 0, & \alpha & \text{ a real number.}\end{aligned}$$