Homework 1

1. Solve the differential equations:

   a. $x\dfrac{dy}{dx} = 4y;$

   b. $y \ln x \dfrac{dx}{dy} = \left(\dfrac{y+1}{x}\right)^2;$

   c. $(e^y + 1)^2 e^{-y} dx + (e^x + 1)^3 e^{-x} dy = 0;$

   d. $\dfrac{dy}{dx} = \dfrac{xy + 3x - y - 3}{xy - 2x + 4y - 8}.$

2. Solve the intial value problem

$$\sqrt{1 - y^2}dx - \sqrt{1 - x^2}dy = 0, \quad y(0) = \frac{\sqrt{3}}{2}.$$

3. Find a solution to the equation

$$x\frac{dy}{dx} = y^2 - y$$

   that passes through the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

4. Find the solution to the initial value problem

$$\frac{dy}{dx} = \frac{2x + 1}{2y}, \quad y(-2) = -1.$$

   Find the exact interval of definition of the solution.