

## Review

1. Solve the differential equation

$$y' = 2xy + 3y - 4x - 6.$$

2. Solve the initial value problem

$$\frac{dy}{dx} = xy^2 - 9x - 2y^2 + 18, \quad y(0) = -5.$$

3. Solve the differential equation

$$\frac{dy}{dx} = xy^2 + 9x + 2y^2 + 18.$$

4. Solve the differential equation

$$\frac{dy}{dx} = \frac{y(y+1)}{x(x-1)}.$$

5. Solve the differential equation

$$(e^y + 1)^2 e^{-y} dx + (e^x + 1)^3 e^{-x} dy = 0.$$

6. Solve the differential equation

$$y' + 5y = 7.$$

7. Solve the differential equation

$$y' + ty = -t^3.$$

8. Solve the initial value problem

$$ty' + 2y = t^2 + t + 1, \quad y(1) = 2.$$

9. Solve the differential equation

$$y' \cos t + y \sin t = 1, \quad -\pi/2 < t < \pi/2.$$

10. Solve the initial value problem

$$y' + e^t y = 2e^t, \quad y(0) = 0.$$

11. For the following differential equation determine if it is exact or not. If it is not exact, see if you can find an integrating factor, then solve it.

$$(3x + 1)dx + (2y + 3)dy = 0.$$

12. For the following differential equation determine if it is exact or not. If it is not exact, see if you can find an integrating factor, then solve it.

$$(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0.$$

13. For the following differential equation determine if it is exact or not. If it is not exact, see if you can find an integrating factor, then solve it.

$$(3xy + y^2) + (x^2 + xy)y' = 0.$$

14. For the following differential equation determine if it is exact or not. If it is not exact, see if you can find an integrating factor, then solve it.

$$y + (2xy - e^{-2y})y' = 0$$

15. For the following differential equation determine if it is exact or not. If it is not exact, see if you can find an integrating factor, then solve it.

$$(x + 2) \sin y + (x \cos y)y' = 0.$$

16. Bob has an ant problem. On the first day of May, Bob discovers he has a small red ant hill in his back yard, with a population of about 100 ants. If conditions are just right red ant colonies have a growth rate of 240% per year during the first four years. If Bob does nothing, how many ants will he have next May? How many in five years?

17. A certain college graduate borrows 8000 dollars to buy a car. The lender charges interest at an annual rate of 10%. Assuming that the interest is compounded continuously and that the borrower makes payments at a constant annual rate  $k$ , determine  $k$  so that the loan is paid in 3 years.

18. Suppose that a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 180° F and the room temperature is 76° F and after five minutes the coffee has temperature 168° F, find how long will it take for the coffee to reach the temperature of 155° F.

19. Solve the dimensionless logistic equation

$$\frac{dx}{dt} = (1 - x)x.$$

Discuss the monotonicity of the solution, the intervals of convexity/concavity, the equilibria and the behavior of the solution as  $t \rightarrow \infty$ .

20. A glass of room-temperature water is carried out onto a balcony from an apartment where the temperature is 24°C. After five minutes the water has temperature 21°C and after ten minutes it has temperature 20°C. What is the outdoor temperature?