

Homework 4

1. In each of the following two problems determine the form of the eigenfunctions and the algebraic equation that determines the eigenvalues. Estimate the eigenvalues that have large absolute values.

$$\begin{aligned}y'' - \lambda y &= 0, & y(0) &= 0, & y(\pi) + y'(\pi) &= 0, \\y'' - \lambda y &= 0, & y(0) &= y'(0) = 0, & y(1) &= 0.\end{aligned}$$

Then normalize the eigenfunctions.

2. In each problem, transform the given equation into the form $(p(x)y')' + q(x)y = 0$:

$$\begin{aligned}x^2 y'' + xy' + (x^2 - \nu^2)y &= 0, & \nu & \text{ a real number,} \\(1 - x^2)y'' - xy' + \alpha^2 y &= 0, & \alpha & \text{ a real number.}\end{aligned}$$