Review 2

1. (a) What is the general form of the heat equation that describes the heat distribution $u(x,t)$ in a rod?

   (b) Write the formula for the solution to the heat equation in a rod of length L subject to the boundary constraints

   $$u(0,t) = 0, \quad u(L,t) = 0, \quad t > 0,$$
   $$u(x,0) = f(x), \quad 0 \le x \le L.$$

   (c) Consider the conduction of heat in a rod 30 cm in length whose ends are maintained at 0°C for all $t > 0$. Assume that $\alpha^2 = 4$. Find the expression of the temperature $u(x,t)$ at time t if the initial temperature is

   $$u(x,0) = \begin{cases} 50, & \text{if } 0 \le x \le 15, \\ 10, & \text{if } 15 \le x \le 30. \end{cases}$$

2. (a) What is the general form of the heat equation that describes the heat distribution $u(x,t)$ in a rod?

   (b) Write the formula for the solution to the heat equation in a rod of length L subject to the boundary constraints

   $$u(0,t) = 0, \quad u(L,t) = 0, \quad t > 0,$$
   $$u(x,0) = f(x), \quad 0 \le x \le L.$$

   (c) Consider the conduction of heat in a rod 4 cm in length whose ends are maintained at 0°C for all $t > 0$. Assume that $\alpha^2 = 1$. Find the expression of the temperature $u(x,t)$ at time t if the initial temperature is

   $$u(x,0) = \begin{cases} 0, & \text{if } 0 \le x \le 1, \\ 10, & \text{if } 1 \le x \le 4. \end{cases}$$

3. (a) What is the general form of the wave equation that describes the vibration of an elastic string?

   (b) Write the formula for the solution of the wave equation for an elastic string of length L subject to the boundary conditions

   $$u(0,t) = 0, \quad u(L,t) = 0, \quad t > 0,$$
   $$u(x,0) = f(x), \quad u_t(x,0) = 0, \quad 0 \le x \le L.$$

   (c) Consider a string of length 10 cm whose ends are held fixed. The string is set in motion with no initial velocity from an initial position

   $$u(x,0) = \begin{cases} x & \text{if } 0 \le x \le 5 \\ 10 - x & \text{if } 10 \le x \le 10. \end{cases}$$

   Find the displacement $u(x,t)$ of the string. (Assume $a^2 = 9$.)

4. (a) What is the general form of the wave equation that describes the vibration of an elastic string?

(b) Write the formula for the solution of the wave equation for an elastic string of length L subject to the boundary conditions

$$u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0,$$
$$u(x, 0) = f(x), \quad u_t(x, 0) = 0, \quad 0 \le x \le L.$$

(c) Consider a string of length 20 cm whose ends are held fixed. The string is set in motion with no initial velocity from an initial position

$$u(x, 0) = \begin{cases} x & \text{if } 0 \le x \le 5 \\ \frac{20}{3} - \frac{x}{3} & \text{if } 5 \le x \le 20. \end{cases}$$

Find the displacement $u(x, t)$ of the string. (Assume $a^2 = 1$.)

5. (a) Write the Laplace equation in two variables.

(b) Write the general form of the solution to the Laplace equation subject to the Dirichlet boundary conditions

$$u(x, 0) = 0, \quad u(x, b) = 0, \quad 0 < x < a,$$
$$u(0, y) = 0, \quad u(a, y) = f(y), \quad 0 < y < b.$$

(c) Solve the Laplace equation when $a = 1, b = 4$ and

$$f(y) = y(4 - y).$$

6. (a) Write the Laplace equation in two variables.

(b) Write the general form of the solution to the Laplace equation subject to the Dirichlet boundary conditions

$$u(x, 0) = 0, \quad u(x, b) = 0, \quad 0 < x < a,$$
$$u(0, y) = 0, \quad u(a, y) = f(y), \quad 0 < y < b.$$

(c) Solve the Laplace equation when $a = 1, b = 4$ and

$$f(y) = \begin{cases} 2y, & \text{if } 0 \le y \le 2, \\ 8 - 2y, & \text{if } 2 \le y \le 4. \end{cases}$$