

Practice problems

1. Prove by contrapositive that if $n \in \mathbb{Z}$ and $7n + 5$ is odd, then n is even.
2. Assume that $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies

$$|f(x) - f(y)| \geq |x - y|$$

for all real numbers x and y . Prove that f is injective.

3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(f(x)) = -x$. Prove that f is injective.
4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying

$$f(f(x)) = x^2 - x + 1.$$

Prove that f is not injective.

5. Prove that there are infinitely many positive integers n such that \sqrt{n} is not an integer.
6. Prove that there do not exist positive integers m and n such that $m^2 - n^2 = 1$.
7. Prove that there do not exist positive integers a, b, c such that each divides the difference of the other two.