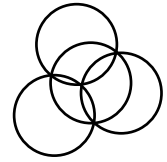


8. DEFINITE INTEGRALS

by Răzvan Gelca



h1. Prove that the function $F : [0, 1) \rightarrow 0, \infty)$,

$$F(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1 - k \cos^2 x}}$$

is increasing.

h2. For what values of $a > 1$ does the integral

$$\int_a^{a^2} \frac{1}{x} \ln \frac{x-1}{32} dx$$

attain its minimal value?

h3. Let p and q be positive numbers. Prove that

$$\int_0^1 (1 - x^p)^{1/q} dx = \int_0^1 (1 - x^q)^{1/p} dx.$$

h4. Let n be a positive integer. Compute the value of the integral

$$I_n = \int_0^\pi \frac{\sin nx}{\sin x} dx.$$

h5. Compute

$$\int_0^\infty \frac{\sin x}{x} dx.$$

h6. Find the integral of the function

$$f(x, y, z) = \frac{x^4 + 2y^4}{x^4 + 4y^4 + z^4}$$

over the unit ball $x^2 + y^2 + z^2 \leq 1$.

h7. Show that for $a, b > 0$,

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \ln \frac{b}{a}.$$