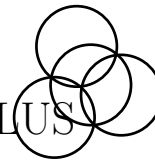


5. PROBLEMS ABOUT DIFFERENTIAL CALCULUS
by Răzvan Gelca



e1. For which positive real numbers a does the equation

$$x^x = a$$

have exactly two distinct positive real solutions?

e2. Prove that the function

$$f(x) = \begin{cases} e^{-1/x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

admits derivatives of any order at 0.

e3. Let $f : [0, 1] \rightarrow \mathbb{R}$ be a continuous function that is twice differentiable on $(0, 1)$ and satisfies $f(0) = f(1) = 0$ and $\min_{x \in [0, 1]} f(x) = -1$. Prove that $\max_{x \in [0, 1]} f''(x) \geq 8$. (Hint. Use the Taylor's Theorem and take the expansion around the minimum of f).

e4. Prove that for all $x \in [-1, 1]$ and for all positive integers n ,

$$(1+x)^n + (1-x)^n \leq 2^n$$