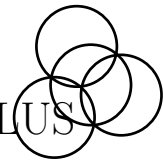


4. PROBLEMS ABOUT DIFFERENTIAL CALCULUS

by Răzvan Gelca



d1. Prove that for all positive real numbers x one has

$$e^x \geq x^e.$$

d2. Find all positive real solutions to the equation

$$2^x = x^2.$$

d3. Let a and b be positive real numbers. Show that for all positive integers n ,

$$(n-1)a^n + b^n \geq na^{n-1}b.$$

d4. Find the minimum of the function

$$f(x) = \frac{(x^2 - x + 1)^3}{x^6 - x^3 + 1}.$$

d5. Prove that not all zeros of the polynomial

$$P(x) = x^4 - \sqrt{7}x^3 + 4x^2 - \sqrt{22}x + 15$$

are real.

d6. Find all real numbers x such that

$$2^x + 5^x = 3^x + 4^x.$$