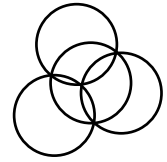


7. DEFINITE INTEGRALS

by Răzvan Gelca



g1. Prove that for every real numbers $a, b \geq 1$

$$\int_0^1 x^a(1-x)^b dx = \int_0^1 x^b(1-x)^a dx.$$

g2. Compute the integral

$$\int_{-1}^1 \frac{\sqrt[3]{x}}{\sqrt[3]{1-x} + \sqrt[3]{1+x}} dx$$

g3. Compute

$$\int_0^{\sqrt{\frac{\pi}{3}}} \sin x^2 + \int_{-\sqrt{\frac{\pi}{3}}}^{\sqrt{\frac{\pi}{3}}} x^2 \cos x^2 dx.$$

g4. Let a, b be positive real numbers. Compute

$$\int_a^b \frac{e^{\frac{x}{a}} - e^{\frac{b}{x}}}{x} dx.$$

g5. Compute the integral

$$\int_0^1 \sqrt[3]{2x^3 - 3x^2 - x + 1} dx.$$

g6. Let $f : [0, 1] \rightarrow \mathbb{R}$ be a continuous functions. Prove that

$$\int_0^\pi x f(\sin x) dx = \pi \int_0^{\frac{\pi}{2}} f(\sin x) dx.$$