

Introduction to Topology – Homework 4

1. Show that if one space is a deformation retract of another, then their fundamental groups are isomorphic. What is the fundamental group of $\mathbb{C} \setminus \{0\}$? What are the covering spaces of $\mathbb{C} \setminus \{0\}$? What are the deck transformations?
2. What is the universal covering space of the annulus $\{z \mid 1 \leq |z| \leq 2\}$?
3. What is the universal covering space of the cartesian product of two spheres $S^2 \times S^2$?
4. The Klein bottle is obtained as a quotient space of $[0, 1] \times [0, 1]$ by the equivalence relations $(s, 0) \equiv (s, 1)$ and $(0, t) \equiv (1, 1 - t)$, for all $s, t \in [0, 1]$. Compute the fundamental group of the Klein bottle.
5. What is the fundamental group of $S^1 \vee S^2$?
6. Let X be the space obtained by gluing $\overline{B^2}$ to S^1 by the map on the boundary $z \rightarrow z^n$. What is the fundamental group of X ?
7. Find a space whose fundamental group is $\mathbb{Z}_3 \times \mathbb{Z}_5$.