Introduction to Topology – Homework 6

1. Let n be a positive integer, and consider the map $f : S^1 \to S^1$, $f(z) = z^n$. Turn the two circles into Δ-complexes so that f becomes a Δ-map. Compute the map induced by it in homology.

2. The first homology group of the Klein bottle is $\mathbb{Z} \oplus \mathbb{Z}_2$. Find a Δ-map from the Klein bottle to the circle that gives rise to the map

$$\mathbb{Z} \oplus \mathbb{Z}_2 \to \mathbb{Z}, \quad (m, n) \mapsto 2m.$$

3. What is the map induced at the level of the second homology group by the continuous map

$$S^1 \times S^1 \to S^1 \times S^1, \quad (z, w) \mapsto (z^2, w^3).$$